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The Effect of Water Index on Water Cut Increasing Rate

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Abstract: Water cut is an important evaluation index which can comprehensively reflects the development effect of a water flooding reservoir. Study the variation law of water cut is very important for reservoir development evaluation. A mathematical model for estimate water cut and water cut increasing rate was established based on Corey expression of relative permeability curve and fraction flow equation. Three parameters were proposed to quantitatively describe the character of water cut increasing rate. The three parameters have specific meanings and can describe the shape character of the relation curve between water cut increasing rate and water cut and the relation curve between water cut increasing rate and recovery percent. The effect of water index on water cut increasing rate was discussed. The relationship curve between three parameters and water index under the condition of different water oil mobility ratio was established. The result shows that when water-oil mobile ratio is greater than 10, water index has great effect on water cut increasing rate. With the increase of water index, the maximum value of water cut increasing rate gradually increase, the recovery degree when water cut increasing rate reaches the maximum value gradually increase and the water cut when water cut increasing rate reaches the maximum value also gradually increase. When water-oil mobile ratio is less than 10, water index has little effect on water cut increasing rate.

Keywords: Water Cut, Water Cut Increasing Rate, Relative Permeability, Water Index

1. Introduction

Water cut is an important evaluation index which can comprehensively reflects the development effect of a waterflooding reservoir. Different reservoirs have different variation characters of water cut. Many reservoir engineers aim to describe the variation law of water cut. Among them, some study water cut by establishing theoretical models [1-7]. And some engineers study water cut by establishing the numerical simulation models. Reservoir simulation prediction can be reasonably accurate, however, some simulation data such as geological, geophysical, reservoir fluid and rock properties, etc. are not readily available. Besides, when a field has huge grid-blocks and a long history of production data, it is very time consuming. In this paper, we aim to study variation characters of water cut by method of establishing the theoretical model. Many engineers proposed many analytical models for predicting oil production and water cut [8-10], There are several models associating water cut with oil production or correlated water cut with cumulative oil production [11-13].

In this paper, we built a mathematical model for estimate water cut and water cut increasing rate based on relative permeability curve and fraction flow equation. With the help of three parameters describing the shape character of the relation curve the relation curve between water cut increasing rate and water cut and the relation curve between water cut increasing rate and recovery percent, the effect of water index on water cut increasing rate was conducted.

2. Mathematical Model of Water Cut Increasing Rate

Relative permeability curve of oil and water phase is the basis of oil and water seepage flow. Relative permeability of oil phase and water phase can be described in many ways, among them Corey expression is the commonly used expression.

$$K_{rw} = K_{rw}(S_{or}) \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \right)^{n_w} \tag{1}$$

$$K_{ro} = K_{ro}(S_{wi}) \left(\frac{1 - S_{or} - S_w}{1 - S_{wi} - S_{or}} \right) \left(\frac{1 - S_{or} - S_w}{1 - S_{wi} - S_{or}} \right)^{n_o} \tag{2}$$

Where K_{rw} is water relative permeability; $K_{rw}(S_{or})$ is oil relative permeability at residual oil saturation; n_w is water index; K_{ro} is oil relative permeability; $K_{ro}(S_{wi})$ is oil relative permeability at irreducible water saturation; S_{wi} is irreducible saturation; n_o is oil index; S_w is average water saturation.

Based on fractional flow equation, water cut can be described as follows:

$$f_w = \frac{Q_w}{Q_l} = \frac{Q_w}{Q_o + Q_w} = \frac{1}{1 + \frac{\mu_w B_w}{\mu_o B_o} \frac{K_{ro}}{K_{rw}}} \tag{3}$$

Where f_w is water cut; μ_w is formation viscosity, $\mathrm{mPa \cdot s}$; B_w is formation water volume factor; B_o is formation oil volume factor.

Combined with Equation (1), Equation (2), Equation (3), water cut can be expressed as:

$$f_w = \frac{\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \right)^{n_w}}{\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \right)^{n_w} + \left(\frac{1 - S_{or} - S_w}{1 - S_{wi} - S_{or}} \right)^{n_o}} \tag{4}$$

According to the definition of recovery percent (R), R can be expressed as follows:

$$R = \frac{S_w - S_{wi}}{1 - S_{wi}} \tag{5}$$

Where R is recovery percent of geologic reservoir.
Equation (4) can be converted into:

$$f_w = \frac{\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_w}}{\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_w} + \left(\frac{1 - S_{or} - S_{wi} - R \left(\frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)}{1 - S_{wi} - S_{or}} \right)^{n_o}} \tag{6}$$

According to the definition of water cut increasing rate, it can be expressed as:

$$f_w' = \frac{df_w}{dR} \tag{7}$$

Where f_w' is water cut increasing rate.
Combined with Equation (6) and Equation (7), water cut increasing rate can be expressed as:

$$f_w' = \frac{\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_w - 1} \left(1 - \frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_o - 1} \left[n_w \left(1 - \frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right) + n_o \frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right]}{\left[\frac{\mu_o B_o K_{rw}(S_{or})}{\mu_w B_w K_{ro}(S_{wi})} \left(\frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_w} + \left(1 - \frac{1 - S_{wi}}{1 - S_{wi} - S_{or}} R \right)^{n_o} \right]^2} \tag{8}$$

3. The Effect of Water Index Water Cut Increasing Rate

3.1. Project Design

According to the equation (8), there are three main factors infecting water cut increasing rate. The three main factors are water-oil mobility ratio, oil index and water index. In this paper, we focus on the water index. In order to study the effect of water index on water cut increasing rate, we designed many projects with different water index on the condition that other influencing factors keeps constant. The end point of relative permeability are as follows: the irreducible water saturation is 0.289, residual oil saturation is 0.215, the oil relative permeability at irreducible water saturation is 1.0, water relative permeability at residual oil saturation is 0.414, the oil index is 2.0.

The water relative permeability with different water index are shown in figure 1. As can be seen form figure 1, with the increase of water index, water relative permeability curve became more and more concave.

The relation curves between water cut (f_w) and recovery percent (R), water cut increasing rate(f_w') and water cut(f_w), water cut increasing rate(f_w') and recovery percent (R) are shown respectively in figure 2, figure 3 and figure 4. As can be seen from figure 2, with the decrease of water index, the relation curve between water cut and recovery percent become more and more convex.
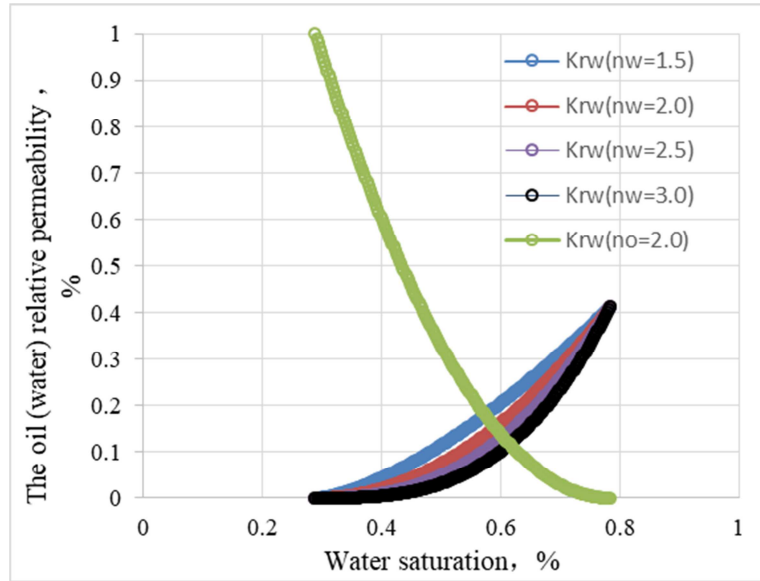
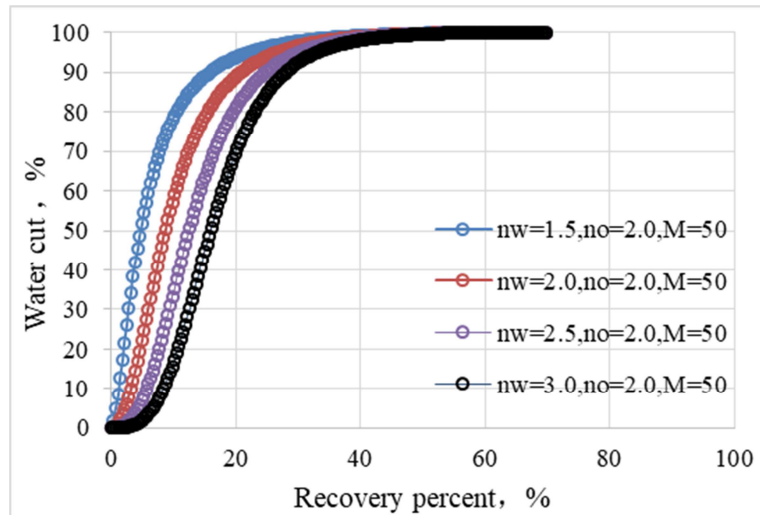*Figure 1.* The relative permeability under different water index.

Figure 2. Relation curve between water cut (f_w') and recovery percent (R) on the condition of different water indexes (n_w).
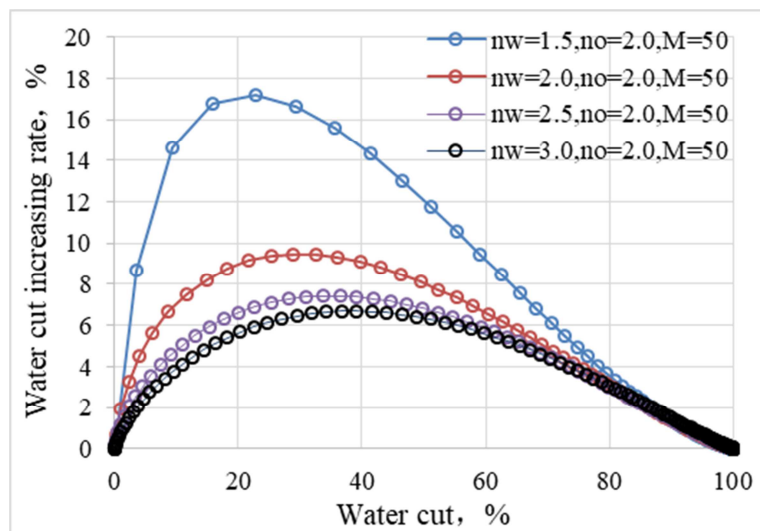

Figure 3. Relation curve between water cut increasing rate (f_w') and water cut (f_w) with different water index (n_w).

As can be seen from figure 3, with the decrease of water index, water cut increasing rate gradually decrease and the curve gradually deviate to the right.
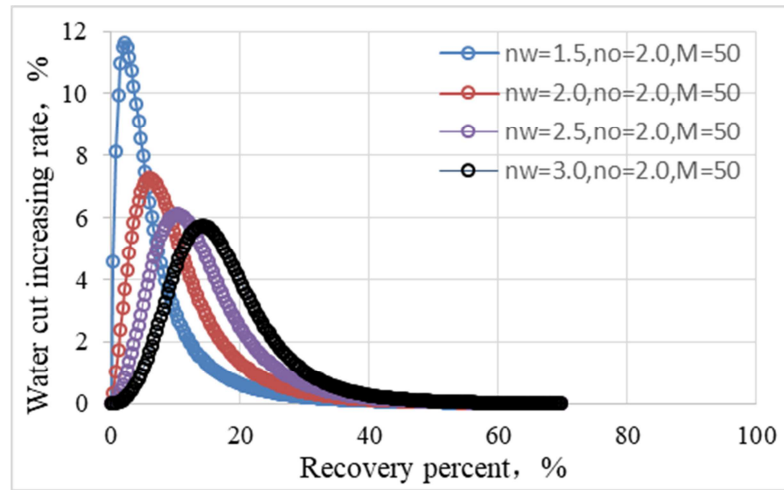

Figure 4. *Relation curve between water cut increasing rate (f_w') and recovery percent (R) with different water indexes (n_w).*

As can be seen from figure 4, with the decrease of water index, the value of water cut increasing rate gradually decrease and the curve gradually deviate to the right.

3.2. The Quantitatively Describe of Water Cut Increasing Rate

In order to quantitatively describe the analytical curve of water cut increasing rate, we proposed three characteristic parameters. The three parameters are the maxima value of water cut increasing rate $f_w'_{max}$, the water cut when water cut increasing rate reaches to the maximum value $f_w(f_w'_{max})$ and the recovery degree when water cut increasing rate reaches to the maximum value $R(f_w'_{max})$.

The maxima value of water cut increasing rate ($f_w'_{max}$), the water cut when water cut increasing rate reaches to the maximum value ($f_w(f_w'_{max})$) and the recovery degree when water cut increasing rate reaches to the maximum value ($R(f_w'_{max})$) on the condition of different water indexes (n_w) are shown in table 1. The relation curve between this three parameters and water index are respectively shown in figure 5, figure 6 and figure 7.

Table 1. *Character parameters of water cut increasing rate (f_w') curve on the condition of different water indexes (n_w).*

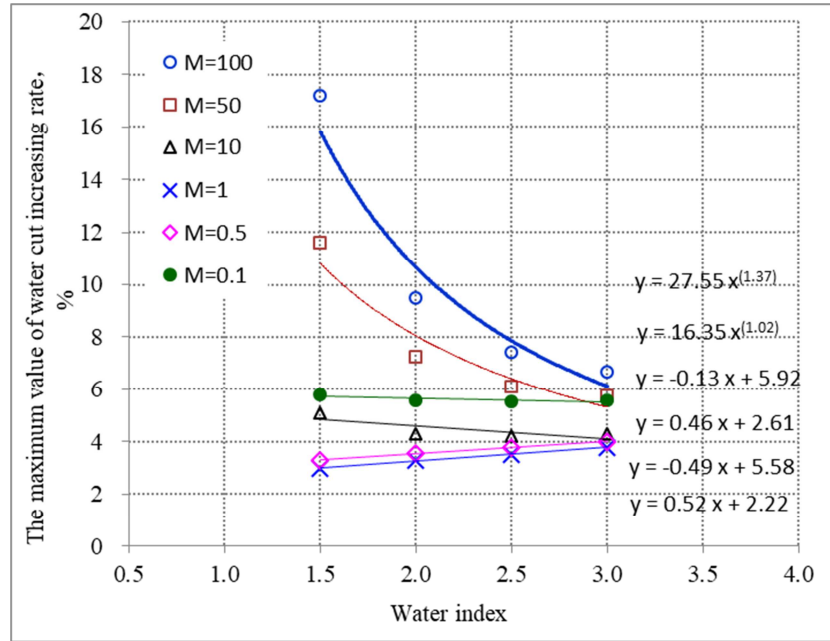| M	n_w	n_o	$f_w'_{max}$	$f_w(f_w'_{max})$%	$R(f_w'_{max})$%
100	1.5	2	17.2	22.8	1.4
100	2.0	2	9.5	28.9	4.2
100	2.5	2	7.4	36.4	8.0
100	3.0	2	6.7	39.2	11.5
50	1.5	2	11.6	21.6	2.1
50	2.0	2	7.3	30.1	5.9
50	2.5	2	6.1	35.4	10.1
50	3.0	2	5.7	38.5	14.0
10	1.5	2	5.1	24.6	6.3
10	2.0	2	4.3	35.5	13.3
10	2.5	2	4.2	40.1	18.5
10	3.0	2	4.3	43.0	22.7
1	1.5	2	3.0	49.1	31.0
1	2.0	2	3.3	51.0	35.2
1	2.5	2	3.5	51.4	38.0
1	3.0	2	3.8	51.3	40.1
0.5	1.5	2	3.3	57.4	41.2
0.5	2.0	2	3.5	56.1	42.9
0.5	2.5	2	3.8	55.8	44.6
0.5	3.0	2	4.0	55.4	46.0
0.1	1.5	2	5.8	68.3	56.9
0.1	2.0	2	5.6	66.0	56.9
0.1	2.5	2	5.5	65.3	57.2
0.1	3.0	2	5.6	64.7	57.6

Figure 5. *Maximum value of water cut increasing rate* $(f_w'{}_{max})$ *on the condition of different water index* (n_w)*.*

As can be seen from figure 5, when the value of water-oil mobile ratio (M) is large than 50, water index (n_w) has great impact on the maximum value of water cut increasing rate ($f_w'{}_{max}$). With the increase of water index (n_w), the maximum value of water cut increasing rate ($f_w'{}_{max}$) rapidly decrease on the condition that the value of water-oil mobile ratio (M) is greater than 10. And the relation between the maximum value of water cut increasing rate and water index is exponential relationship. When the value of water-oil mobile ratio (M) is smaller than 10, water index (n_w) has no significant impact on the maxima value of water cut increasing rate ($f_w'{}_{max}$) and the relation between the maximum value of water cut increasing rate and water index is linear relationship.
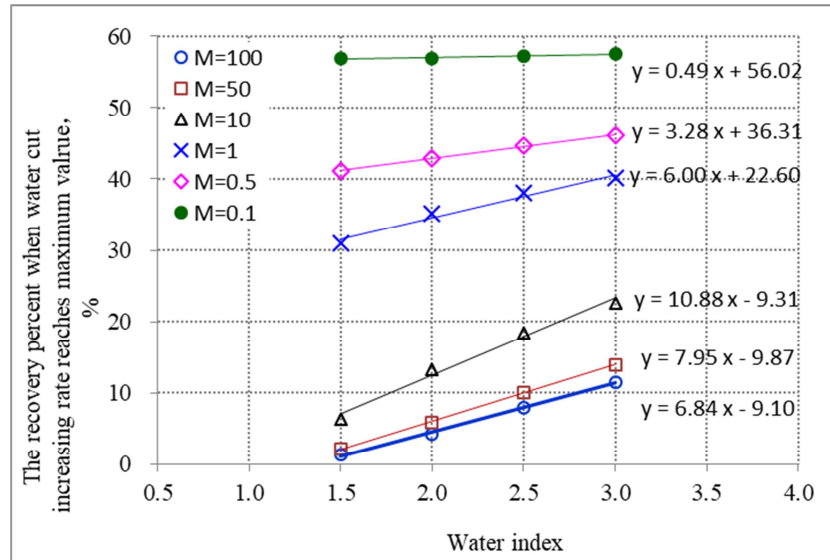

Figure 6. *The relation between water index* (n_w) *and* $R(f_w'{}_{max})$*.*

As can be seen from figure 6, with the increase of water index (n_w), the recovery degree when water cut increasing rate reaches to the maximum value ($R(f_w'{}_{max})$) gradually increase and there is a linear relationship between them. When the value of water oil ratio approximately equals to 10, water index has great impact on the time when water increasing rate reaches maximum value. When the value of water oil ratio approximately is less than 0.5, water index has less impact on the time when water increasing rate reaches maximum value.
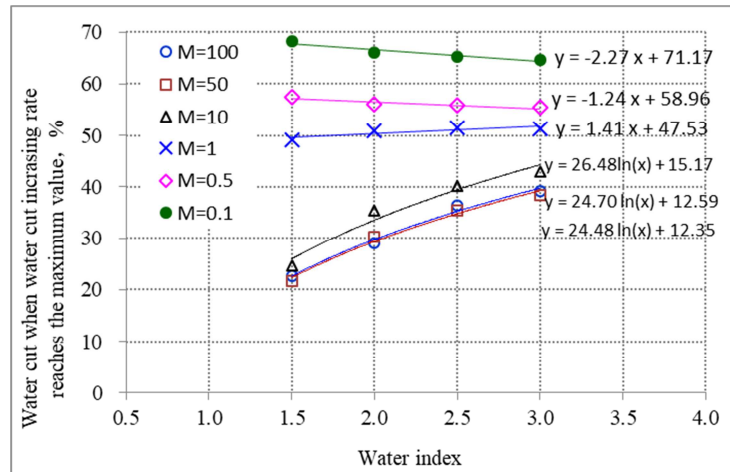
Figure 7. The relation between n_w and $f_w(f_w'_{max})$.

As can be seen form Figure 7, with the increase of water index, the water cut when water cut increasing rate reaches to the maximum value ($f_w(f_w'_{max})$) gradually increase and there is a linear relationship between them. When the value of water oil ratio equals to 1, water index has less impact on the water cut when water cut increasing rate reaches to the maximum value ($f_w(f_w'_{max})$).

Conclusions obtained by figure 5 ~figure 7 are as follows: when the value of water oil ratio large greater than or equal to 10, water index has little effect on water cut increasing rate.

4. Conclusions

A mathematical model for estimate water cut and water cut increasing rate was established. According the mathematical model, water cur increasing rate was effected by recovery percent, water-oil mobility ratio, water index and oil index of relative permeability.

Three parameters were proposed to quantitatively describe the character of water cut increasing rate. The three parameters are the maxima value of water cut increasing rate, the water cut when water cut increasing rate reaches to the maximum value and the recovery degree when water cut increasing rate reaches to the maximum value.

The effect of water index and oil index on water cut increasing rate was discussed. When water-oil mobile ratio is greater than 10, water index has great effect on water cut increasing rate. With the increase of water index(n_w), the maxima value of water cut increasing rate ($f_w'_{max}$) gradually increase and the recovery degree when water cut increasing rate reaches to the maximum value ($R(f_w'_{max})$) gradually increase.

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