

# Calculation of Relative Permeability Curve Based on Type C Water Drive Curve

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**Abstract:** The oil-water relative permeability curve is the basis for the study of oil-water two-phase flow, and is indispensable for the calculation of oilfield development parameters, dynamic analysis and reservoir numerical simulation. In the absence of oil-water relative permeability curve, how to obtain the oil-water relative permeability curve with high accuracy that can represent the percolation characteristics of the whole area and ensure the reliability of the obtained data has very important practical significance, which is the premise for carrying out the above research work. The characteristic curve of type C water drive belongs to the linear relationship of cumulative output, which has the advantages of strong stability and more reliable results compared to other water drive curves. In this paper, oil-water relative permeability curve is calculated based on C-type water drive curve, and the formula for calculating relative permeability curve using C-type water drive curve is derived in detail, and the specific calculation steps are given. The reliability of the obtained relative permeability curve is verified by examples. Through actual data verification, the oil-water relative permeability curve obtained by the method in this article takes into account the heterogeneity of the reservoir, conforms to the percolation characteristics of the oilfield, and is basically consistent with the actual production situation, which has certain practicality.

**Keywords:** Relative Permeability Curve, C-Type Water Drive Curve, Percolation Characteristics

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## 1. Introduction

Relative permeability curve is an important basic data in reservoir engineering research, which is applied to many aspects of reservoir engineering research. At present, the relative permeability curve can be obtained by experimental and non-experimental methods. The experimental method mainly uses cores to measure the relative permeability curve. The experimental results are affected by factors such as core heterogeneity, wettability, fluid saturation history, etc. Even if the cores are obtained in the same oil layer, the measured relative permeability curves may be significantly different [1]. The non-experimental method includes logging data estimation method, capillary pressure curve method, empirical formula method, and production data calculation method [2-5]. The first three methods have limitations and low accuracy issues. The method of using production data to calculate oil-water relative permeability curve considers the actual production situation of the oilfield and can reflect the overall percolation characteristics of heterogeneous reservoirs. The relative

permeability curve calculated from production data is mainly obtained from production performance data or water drive characteristic curve, which has been discussed by many scholars [6-16].

The curve of type C water drive belongs to a linear relationship of cumulative output, and the trend of cumulative output is not affected by short-term fluctuations. It has strong stability and the highest correlation coefficient and accuracy when fitting and calculating development index [17-19]. This paper obtains an oil-water relative permeability curve with high accuracy and coinciding with the percolation characteristics of the oil field through the C-type water drive curve to solve the problem that the experimental results of cores have differences with each core, and some low permeability cores or oil fields are not easy to measure or have not carried out the measurement of the relative permeability curve due to geological and development factors.

## 2. Formula Derivation

The relationship equation of the C-type water drive curve is

[20]:

$$\frac{L_P}{N_P} = a + bL_P \quad (1)$$

In the above equation,  $L_P$  is the cumulative liquid production, unit:  $10^4\text{m}^3$ ;  $N_P$  is the cumulative oil production, unit:  $10^4\text{m}^3$ ;  $a$  and  $b$  are constants, respectively.

According to formula (1) above, perform linear fitting on the cumulative liquid production and cumulative oil production data to obtain coefficients  $a$  and  $b$ .

According to Darcy's law of oil-water two-phase steady flow, the water saturation is proportional to the oil recovery degree, and the relationship is as follows:

$$R = \frac{S_w - S_{wi}}{1 - S_{wi}} \quad (2)$$

Where  $S_w$  is the water saturation, %;  $S_{wi}$  is the irreducible water saturation, %.

The relationship between cumulative oil production and oil recovery degree is as follows:

$$N_P = NR \quad (3)$$

Where  $R$  is the degree of recovery, %;  $N$  is the original oil in place, with a unit of  $10^4\text{m}^3$ .

Based on formulas (1), (2), and (3), the following formula can be derived:

$$S_w = \frac{L_P(1 - S_{wi})}{N(a + bL_P)} + S_{wi} \quad (4)$$

Taking the derivative on both sides of formula (1) and combining it with formula (2) and (3), the relationship between water cut and water saturation can be obtained:

$$f_w = \frac{a - \left[ 1 + \frac{bN}{1 - S_{wi}} \cdot S_{wi} - \frac{bN}{1 - S_{wi}} \cdot S_w \right]^2}{a} \quad (5)$$

Assuming  $A = 1 + \frac{bN}{1 - S_{wi}} \cdot S_{wi}$ ,  $B = \frac{bN}{1 - S_{wi}}$ ,  $C = a$ , By organizing the above equation, the following formula can be obtained:

$$f_w = \frac{C - [A - B \cdot S_w]^2}{C} \quad (6)$$

In homogeneous and equal thickness oil layers, the production formula of oil-water two-phase plane radial flow is as follows when gravity and capillary pressure are ignored:

$$Q_i = \frac{2\pi K K_{ri} h \Delta P}{\mu_i B_i \ln(r_e/r_w)} \quad (7)$$

Where  $K$  is the absolute permeability;  $Q_i$  is the water or oil production,  $\text{m}^3/\text{d}$ ;  $K_{ri}$  is the Relative permeability of oil or water phase, dimensionless;  $h$  is the thickness of the oil layer, m;  $\Delta P$  is the production pressure difference, MPa;  $\mu_i$  is the viscosity of oil or water;  $B_i$  is the volume coefficient of oil or water, dimensionless;  $r_e$  is the radius of the oil drainage area, m;  $r_w$  is the radius of the oil well, m.

According to formula (7), the relationship formula for water cut can be obtained as follows:

$$f_w = \frac{Q_w}{Q_o + Q_w} = \frac{1}{1 + \frac{\mu_w B_w}{\mu_o B_o} \cdot \frac{K_{ro}}{K_{rw}}} \quad (8)$$

Where  $B_o$  is the oil volume coefficient, %;  $B_w$  is the formation water volume coefficient, %;  $\mu_o$  and  $\mu_w$  are the crude oil viscosity and formation water viscosity, unit:  $\text{mPa} \cdot \text{s}$ ;  $K_{ro}$  is oil phase Relative permeability, dimensionless;  $K_{rw}$  is the water phase Relative permeability, dimensionless;  $Q_w$  and  $Q_o$  are daily water and oil production, unit:  $\text{m}^3/\text{d}$ .

According to formula (8), the relationship between oil-water relative permeability ratio and water cut can be obtained as follows:

$$\frac{K_{ro}}{K_{rw}} = \frac{\mu_o B_o}{\mu_w B_w} \cdot \left( \frac{1}{f_w} - 1 \right) \quad (9)$$

Assuming  $D = \frac{\mu_o B_o}{\mu_w B_w}$ , and substituting formula (6) into formula (9), the relationship between oil-water relative permeability ratio and water saturation can be obtained:

$$\frac{K_{ro}}{K_{rw}} = D \left[ \frac{C}{C - (A - B \cdot S_w)^2} - 1 \right] \quad (10)$$

The values of  $A$ ,  $B$ ,  $C$  and  $D$  can be obtained from the basic data of the reservoir. The water saturation under the cumulative liquid production at different times can be calculated according to formula (4), and then the oil-water relative permeability ratio under the water saturation can be calculated according to formula (10).

Generally, the function of oil-water relative permeability is usually expressed in the form of power function [21], and the expressions are:

$$K_{ro} = K_{ro}(S_{wi}) \left( \frac{1 - S_{or} - S_w}{1 - S_{wi} - S_{or}} \right)^m \quad (11)$$

$$K_{rw} = K_{rw}(S_{or}) \left( \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \right)^n \quad (12)$$

Where  $S_{or}$  is the residual oil saturation, dimensionless;  $m$ ,

$n$  are constant.  $K_{rw}(S_{or})$  and  $K_{ro}(S_{wi})$  are respectively the water phase relative permeability under residual oil saturation and the relative permeability of oil phase under irreducible water saturation, the value of the oil relative permeability under irreducible water saturation can be 1.

Assuming  $y = \lg \frac{K_{ro}}{K_{rw}}$ ,  $\alpha = \lg \left[ \frac{K_{ro}(S_{wi})}{K_{rw}(S_{or})} \right]$ ,  
 $x_1 = \lg \left( \frac{1 - S_{or} - S_w}{1 - S_{wi} - S_{or}} \right)$ ,  $x_2 = \lg \left( \frac{S_w - S_{wi}}{1 - S_{wi} - S_{or}} \right)$ , the equation as follow can be obtained:

$$y = \alpha + mx_1 - nx_2 \quad (13)$$

After the oil-water relative permeability ratio at different times is obtained from Formula (10), the binary linear regression is performed from Formula (13) to obtain the values of  $\alpha$ ,  $m$  and  $n$ .

The water phase relative permeability under residual oil saturation can be calculated by the following function:

$$K_{rw}(S_{or}) = \frac{1}{10^\alpha} \quad (14)$$

Substitute the value of  $\alpha$  into equation (14) and calculate  $K_{rw}(S_{or})$ , and then calculate the oil phase and water phase relative permeability data under different saturations from equations (11) and (12), and finally obtain the oil-water relative permeability curve.

### 3. Calculation Steps

The oil-water relative permeability is the basis for the study of oil-water two-phase flow, and is indispensable for the calculation of oilfield development parameters, dynamic analysis and reservoir numerical simulation. The steps to calculate the relative permeability curve by using type C water drive curve are as follows:

1. Fit according to the C-type water drive curve and calculate the  $a$  and  $b$  coefficients.

Collect data on accumulated liquid production, accumulated oil production, original oil in place, crude oil volume coefficient, formation water volume coefficient, formation crude oil viscosity, irreducible water saturation, residual oil saturation, etc. in the research area. Organize the data of accumulated liquid and oil production, perform linear fitting on  $L_p$  and  $N_p$  according to formula (1), and regress the coefficients  $a$  and  $b$ .

2. Calculate the water saturation under different liquid production rates:

Substitute the  $a$  and  $b$  coefficients calculated in the previous step into the relationship equation between water saturation and liquid production, and calculate the water saturation values under different liquid production rates.

3. Calculate the constant terms  $A$ ,  $B$ ,  $C$ , and  $D$ , and calculate the oil-water relative permeability ratio under different water saturation based on the relationship between oil-water relative permeability ratio and water

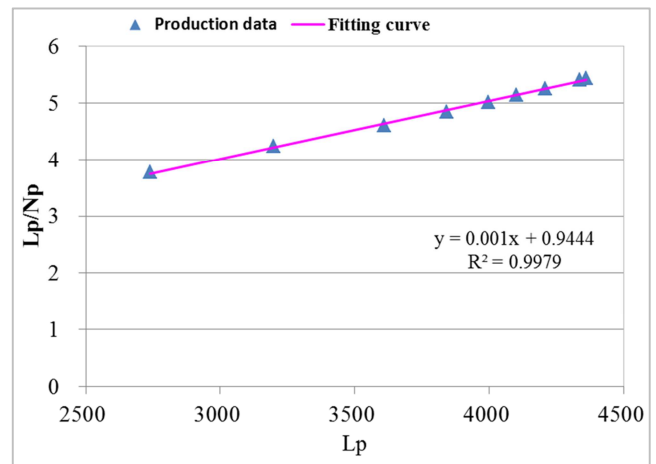
saturation.

4. Calculate water saturation under the cumulative liquid production at different times and oil-water relative permeability ratio under corresponding water saturation based on formula (4) and formula (10);
5. Derive the two sides of formula (11) and (12) respectively, and then establish binary linear equation, conduct binary linear regression, and calculate the coefficients  $m$ ,  $n$  and  $\alpha$ ;
6. Substitute the coefficient of step 5 into equations (11) and (12) to calculate the relative permeability  $K_{ro}$  and  $K_{rw}$  values of oil and water phases under each water saturation.

### 4. Examples

An offshore oil field in the Pearl River Mouth Basin is developed with natural energy, and the edge and bottom water are active. The study area conforms to the characteristics of the C-type water drive curve. There is no laboratory measured data of relative permeability curve in this oilfield. How to use other data to obtain relative permeability curve has become the primary problem for subsequent reservoir engineering and numerical simulation in the study area.

The relevant parameters of the research block are as follows:  $N=2126.46 \times 10^4 \text{m}^3$ ,  $\mu_w=0.36 \text{ mPa} \cdot \text{s}$ ,  $\mu_o=2.16 \text{ mPa} \cdot \text{s}$ ,  $B_o=1.064$ ,  $B_w=1.028$ ,  $S_{or}=0.32$ ,  $S_{wi}=0.351$ . Select the production data related to the production decline stage of the oil reservoir, and fit the ratio of cumulative liquid production to cumulative oil production to cumulative oil production according to equation (1). The fitting results are shown in figure 1.



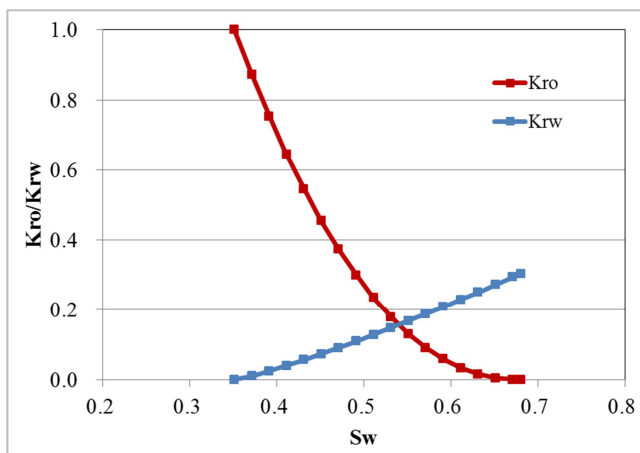
**Figure 1.** Relationship curve between the ratio of cumulative liquid production to cumulative oil production and cumulative oil production.

According to the fitting results, the coefficients  $a$  and  $b$  are 0.9444 and 0.001, and the values of  $A$ ,  $B$ ,  $C$ , and  $D$  are 2.15, 3.27, 0.9444, and 6.21, respectively. Then the water saturation and oil-water relative permeability ratio of the corresponding years are calculated according to formulas (4) and (10).

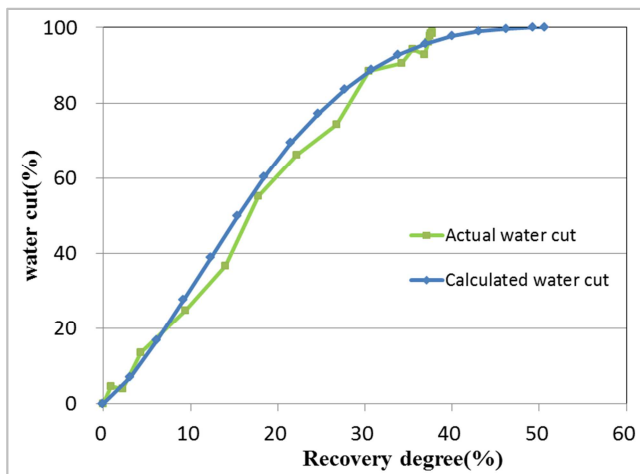
Calculate the  $x_1$  and  $x_2$  values corresponding to different water saturations based on water saturation, irreducible water saturation, and residual oil saturation, and then perform binary

linear fitting on  $x_1$  and  $x_2$  according to formula (13) to obtain the parameters  $\alpha=0.5184$ ,  $m=2.1754$ ,  $n=1.1931$ . The water relative permeability under residual oil saturation is 0.303 obtained from equation (14), then calculates the oil-water relative permeability data according to formula (11) and formula (12), and draws the relative permeability curves, as figure 2.

In order to verify whether the oil-water relative permeability curve obtained by the method in this paper conforms to the actual seepage characteristics, the oil-water relative permeability curve obtained by this method is used to calculate the water cut under different recovery degrees and compare with the actual production water cut. If the calculated water cut rise law is in good agreement with the actual production, this method is correct and feasible.



**Figure 2.** Oil water relative permeability curve calculated Based on Type C Water Drive Curve.



**Figure 3.** Relationship curve between water cut and recovery degree of the offshore oil field.

Calculate the relationship curve between water cut and recovery degree in this example using formulas (2) and (8). The green line in figure 3 shows the relationship curve between the actual water cut and recovery degree, while the blue line is the relationship curve between the water cut and recovery degree calculated by the method in this paper. From figure 3, it can be seen that the water cut increasing law calculated by the relative permeability curve is in good

agreement with the oil field water cut increasing law, and the two trends are basically consistent, proving the reliability of the relative permeability curve calculated by this method.

## 5. Conclusions

When applying the C-type water drive curve, it is necessary to calculate the values of  $a$  and  $b$  based on the data after the appearance of the straight line segment to ensure the reliability of the results.

For reservoirs that conform to the type C water drive law, the oil-water relative permeability curve can be calculated by the method in this paper, and the calculation results are consistent with the seepage characteristics and production situation of the actual oil field.

The calculation of two-phase relative permeability curve based on the Type C water drive curve of the oil field takes the oil reservoir as a system, which not only reflects the properties of the actual rock and fluid, but also considers the heterogeneity of the actual oil reservoir. The oil-water two-phase relative permeability curve obtained based on C-type water drive curve can represent the percolation characteristics of the entire oil reservoir.

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## Biography

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